J. Fluid Mech. (1977), *vol.* 80, *part* 1, *pp.* 17–31
Printed in Great Britain

A study of transonic flow around spheres

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(Received 6 July 1976 and in revised form 9 November 1976)

The paper presents a new analytical and experimental study of transonic flow around spheres. The results of the analytical study, which employs the method of orthogonal collocation for simultaneous solution of the momentum equations, the equation of continuity and the energy equation, are compared with hitherto unpublished measurements obtained on spheres of various sizes (1·02, 2·54 and 3·81 cm in diameter) in air, in dry steam and in wet steam with free-stream Mach numbers in the transonic range ($0{\cdot}58 < M_\infty < 0{\cdot}97$). The relationship $\theta_{\mathrm{sh}} = 91{\cdot}78 + 8{\cdot}59\ M_\infty$ between the attached-shock angle and the free-stream Mach number was obtained by fitting the theoretical pressure distributions to the experimental ones.

1. Introduction

The problem of axisymmetric flow of a compressible medium around blunt bodies has occupied a central position in the field of fluid dynamics in recent years. This can be traced to the practical importance of bodies of revolution in missile and launching vehicle aerodynamics, in aeroplane problems and in space re-entry problems. But the major portion of the research reported to date in this area is concerned only with flows that are either low subsonic or supersonic, and relatively little work has been published on transonic flows. It seems obvious, and we hope to show this in what follows, that a need exists for a simplified analytical solution of transonic flow around blunt bodies and that there is a definite lack of experimental data for such flows. To alleviate the situation we have studied transonic flow ($0{\cdot}58 < M_\infty < 0{\cdot}97$) around spheres, both analytically and experimentally, and some of our findings are reported in this paper.

For $M_\infty < 1$ the flow past a sphere may be characterized with reference to critical values of the Reynolds and Mach numbers as follows. If the sphere is large, Re_{crit} will be reached before M_{crit} when the flow velocity is increased from zero. While $Re < Re_{\mathrm{crit}}$ the boundary layer on the sphere will separate laminarly at about 82°, this separation point continuously shifting downstream when Re is increased just beyond its critical value. On further increasing U_∞, M_{crit} is reached. This signifies the start of conditions for which a supersonic pocket makes its appearance adjacent to the sphere, terminating in a shock near 90°. Interaction of this shock with the boundary layer causes the latter to separate at about the position of the shock, thus moving the point of separation upstream from its previous position and thereby increasing the wake to nearly the size it had when *the boundary layer separated laminarly*.

If the sphere is small M_{crit} is reached upon increasing the velocity while Re is still less than Re_{crit}. Boundary-layer separation will take place in this case at the position of the shock, at about 90°, owing to the interaction of the shock with the boundary layer. The size of the resulting wake is almost the same as for $Re < Re_{\mathrm{crit}}$ and $M_\infty < M_{\mathrm{crit}}$. A further increase in the Reynolds number has virtually no effect on the flow pattern, as the point of separation is fixed by the position of the shock even though the flow in the boundary layer might be turbulent.

Theoretical solution of transonic flow presents quite a formidable problem. Kaplan (1940) found a solution for subsonic flow ($M_\infty < 0{\cdot}5$) around a sphere using the Rayleigh–Janzen method. Terms up to M_∞^4 were considered.

Variational principles for irrotational compressible flow were first derived by Bateman (Serrin 1959) and have been used by Wang (1948), Lush & Cherry (1956), Gispert (1957), Rasmussen & Heys (1974) and others to obtain solutions of subsonic flow around cylinders and other bodies. Wang & Santos (1951) used the variational method to find a solution for subsonic flow around a sphere at $M_\infty = 0{\cdot}5$. In spite of taking $\gamma = 2$ for air, the agreement with the Rayleigh–Janzen method of solution was remarkably good. Lin & Rubinov (1948) formulated a variational principle for rotational two-dimensional and axisymmetric flow. On the basis of this principle and Wang & Chou's (1951) work, Fizdon (1962) concluded that variational methods are promising for calculation of transonic flow. Rasmussen's (1974) review article on the application of variational methods in compressible flow calculations is very informative.

Only three papers, to our knowledge, have attempted a solution of transonic flow around blunt bodies. Lipnitskii & Lifshits (1970) used a time-asymptotic finite-difference method to solve the non-isentropic Euler equations and exact boundary conditions for closed blunt bodies. South & Jameson (1973) presented a finite-difference relaxation method for numerical solution of the full potential equation and exact boundary conditions for general axisymmetric bodies (blunt or pointed nose). This method is also applicable to a supersonic free stream. However, these two methods were tried mostly on geometries other than spherical; solution for a sphere was attempted at $M_\infty = 0{\cdot}8$ only. Furthermore no comparison with experimental data was made. Agreement between the two methods for both the pressure distribution and the sonic line was poor. No attempt was made to investigate other Mach numbers. For transonic flow around a sphere South & Jameson concluded, "It would be interesting and valuable to have a third independent calculation for these flow fields, preferably with Euler's equations, to pin down with confidence the magnitude of the errors and distortion stemming from the isentropic assumption and lack of shock fitting." Hsieh (1975) has used the method developed by South & Jameson for transonic flow ($0{\cdot}7 < M_\infty < 1$) around a hemispherical cylinder, but the agreement between theoretical and experimental pressure distributions is not satisfactory, nor do his theoretical flow-field predictions agree with flow-field shadowgraphs. However results at $M_\infty > 1{\cdot}05$ obtained from the computer programs developed by Aunaper (1970) compared well with the experimental results.

The *earlier experimental investigations* were mostly conducted to obtain information about the drag coefficients for spheres. Wieselberger's (1922) report was followed by papers by Bacon & Reid (1924), Fage (1936) and Charters & Thomas (1945). Later publications are those of Maxworthy (1969), Bailey & Hiatt (1970) and Achenbach (1968). Bailey & Hiatt measured the drag in the Mach number range 0·1–6·0 and the Reynolds number range 20–10^5. The tests were conducted in a ballistic range. Achenbach presented drag curves as well as information on the angular position of the separation point as a function of Reynolds number for a smooth sphere. Hundstat (1953) developed a spherical probe for a low subsonic Mach number (0·2) and later Lee & Ash (1956) provided calibration curves for a three-dimensional spherical Pitot probe for Mach numbers up to 0·4. Naumann (1953) measured the drag coefficient of spheres in the domain $10^5 < Re < 6\times 10^5$, $0{\cdot}3 < M < 0{\cdot}9$ of parameter space. He has shown the Reynolds number effect to be negligible when $M > 0{\cdot}7$. Even at M_{crit} this effect is not discernible in the range $5\times 10^4 < Re < 4{\cdot}5\times 10^5$.

Heberle, Wood & Goodrum (1950) and Goodrum & Wood (1951) have reported interferograms for supersonic flow around spheres in the range of free-stream Mach numbers 1·17–1·81. These two studies provide information regarding the location of the detached shock wave as well as the flow field behind the shock. In a recent study Hsieh (1975) reported tests on a hemispherical cylinder 1·0 in. in diameter and 10·0 in. long in the range of free-stream Mach numbers 0·8–1·3; schlieren photographs and pressure distributions were presented.

2. Theory

In the present formulation of the problem of transonic flow around a sphere the following assumptions are made.

(i) The flow is parallel and uniform at infinity.

(ii) The flow is axisymmetric and steady.

(iii) The fluid is inviscid and in thermodynamic equilibrium and the effects of heat conduction and radiation on the flow field are negligible.

(iv) The solid surface is non-conducting.

(v) The flow is isoenergic, i.e.

$$\frac{\gamma}{\gamma-1}\frac{p}{\rho}+\tfrac{1}{2}q^2 = C_3, \quad C_3 = \text{constant}.$$

When the free-stream Mach number M_∞ is in the transonic range a supersonic pocket forms adjacent to the sphere. This pocket is bounded in the downstream direction by a shock wave which extends outwards from the solid surface and bounded in the radial direction by the position 1–2 diameters from the surface where the local Mach number is unity (Heberle *et al.* 1950). In the region behind the shock, extending to infinity in the downstream direction and bounded laterally by the stream surface passing through the position where the shock terminates, the flow is rotational and of non-uniform entropy. This rotational flow region is denoted by D_2 while D_1 is the irrotational flow region.

The differential equations satisfied by the flow variables are as follows (relative to a spherical polar co-ordinate system).

(i) The continuity equation

$$(\rho u r^2 \sin\theta)_{,r} + (\rho v r \sin\theta)_{,\theta} = 0. \tag{1}$$

(ii) The momentum equation

$$\left.\begin{aligned} uu_r + \frac{vu_\theta}{r} - \frac{v^2}{r} &= -\frac{p_r}{\rho}, \\ uv_r + \frac{vv_\theta}{r} + \frac{uv}{r} &= -\frac{p_\theta}{\rho}. \end{aligned}\right\} \tag{2}$$

(iii) The energy equation

$$uS_r + vr^{-1}S_\theta = 0. \tag{3}$$

(iv) The equation of state

$$p = \begin{cases} C_1\rho & \text{in} \quad D_1, & (4a) \\ C_2 \exp(S/Cv) & \text{in} \quad D_2. & (4b) \end{cases}$$

The relevant boundary conditions are

$$U = 0 \quad \text{at} \quad r = r_0, \tag{5a}$$

$$\lim_{r\to\infty} (\rho, u, v) = (\rho_\infty, u_\infty, v_\infty) \quad \text{in} \quad D_1, \tag{5b}$$

$$\lim_{r\to\infty} p = p_\infty \quad \text{in} \quad D_2. \tag{5c}$$

Across the shock, where regions D_1 and D_2 have a common boundary, the continuity equation holds good. To locate this unknown common boundary between regions D_1 and D_2 we treat the shock angle θ_{sh} as a parameter and optimize its value by fitting theoretical pressure distributions to experimental ones. In such a scheme theoretical flow predictions for D_2 would be meaningless owing to the simplicity of our model and we dispense completely with this region.

The continuity equation (1) can be eliminated from further consideration by the introduction of the stream function $\Psi(r,\theta)$:

$$\frac{u}{U_\infty} = \frac{1}{\rho r^2 \sin^2\theta}\Psi_\theta, \quad \frac{v}{U_\infty} = -\frac{1}{\rho r \sin\theta}\Psi_r. \tag{6}$$

Equations (1)–(3), (4a), (5a) and (6) were simplified to a system of two simultaneous nonlinear partial differential equations in Ψ and ρ. These were then, in our first trial, reduced by the Galerkin–Kantorovich method and the ordinary differential equations that resulted were solved by Haming's modified predictor–corrector method. We have encountered difficulties with this solution scheme for two reasons: (a) the domain of initial values which give stable solutions to the equations is small and (b) for higher-order Galerkin approximations the algebra proves to be far too taxing owing to the presence of the $\rho^{\gamma+1}$ term in the equations.

Orthogonal collocation was tried and finally accepted. For the application of this method the flow field was made finite through the transformation

$$z = r_0/r, \quad \alpha = \theta - \pi.$$

In terms of the new variables (1), (2), (4a) and (6) reduce to

$$\left.\begin{aligned} z^6\Psi_z^2 + z^4\Psi_\alpha^2 - 2\sin^2\alpha(a_1\rho^2 - a_2\rho^{\gamma+1}) = 0,\\ \rho(z^2\Psi_{zz} + 2z\Psi_z + \Psi_{\alpha\alpha} - \Psi_\alpha\cot\alpha) - z^2\rho_z\Psi_z - \Psi_\alpha\rho_\alpha = 0.\end{aligned}\right\} \tag{7}$$

Here
$$a_1 = \frac{C_3}{U_\infty^2}, \quad a_2 = \frac{\gamma}{\gamma-1}\frac{\rho_\infty}{\rho_\infty U_\infty^2}$$

and Ψ and ρ have been made dimensionless with $r_0\rho_\infty U_\infty$ and ρ_∞ respectively.

The boundary conditions (5a) in terms of Ψ, ρ, z and α are

$$\Psi(1,\alpha) = \Psi_\alpha(1,\alpha) = 0, \quad \Psi(0,\alpha) = \Psi_0, \quad \Psi(z,0) = 0, \tag{8}$$

where $\Psi_0 = 0{\cdot}5\ \sin^2\alpha\ (z^{-1} - z)$ is the incompressible stream function for flow around a sphere of radius $z = 1$.

It was then assumed that both Ψ and ρ can be expanded in series of orthogonal polynomials. Thus the Nth approximations to the solutions Ψ and ρ are written as

$$\left.\begin{aligned} \Psi_N &= \Psi_0 + (z-z^2)\sin^2\alpha \sum_{i=1}^{N+1}\sum_{j=1}^{N+1} a_{ij}P_{i-1}(z)\,Q_{j-1}(\alpha^2),\\ \rho_N &= 1 + z^2\sum_{i=1}^{N+1}\sum_{j=1}^{N+1} b_{ij}R_{i-1}(z)\,S_{j-1}(\alpha^2).\end{aligned}\right\} \tag{9}$$

Here N^2 is the number of interior collocation points while $(z-z^2)$, $\sin^2\alpha$ and z^2 are weighting functions, which make these expressions satisfy the boundary conditions (8).

The orthogonal polynomials in (9) are defined as follows:

$$P_1(z) = R_1(z) = Q_1(\alpha^2) = S_1(\alpha^2) = 1, \tag{10a}$$

$$\left.\begin{aligned} &\int_0^1 (z-z^2)\,P_j(z)\,P_i(z)\,dz = 0 &(10b)\\ &\int_0^1 z^2R_j(z)\,R_i(z)\,z^2dz = 0 &(10c)\\ &\int_0^{\pi-\theta_1} \sin^2\alpha\,Q_j(\alpha^2)\,Q_i(\alpha^2)\sin\alpha\,d\alpha = 0 &(10d)\\ &\int_0^{\pi-\theta_1} S_j(\alpha^2)\,S_i(\alpha^2)\sin\alpha\,d\alpha = 0 &(10e)\end{aligned}\right\}\ \text{if } i \neq j, \quad i,j = 1,2,3,\ldots,N.$$

The trial functions for Ψ and ρ are substituted into (7) and the residuals set equal to zero at the collocation points $(z_i;\ \alpha_j)$, $i,j = 1, 2, 3, \ldots, N+1$. This provides us with $2(N+1)^2$ conditions for the determination of the $2(N+1)^2$ unknown coefficients a_{ij} and b_{ij}:

$$z_i^6\left(\frac{\partial\Psi}{\partial z}\right)_{ij}^2 + z_i^4\left(\frac{\partial\Psi}{\partial\alpha}\right)_{ij}^2 - 2\sin^2\alpha_j\,(a_1\rho_{ij}^2 - a_2\rho_{ij}^{\gamma+1}) = 0 \tag{11}$$

Probe diameter	Free-stream Mach number, M_∞		
	Air	Wet steam	Dry steam
1·02 cm	0·594	0·59	0·772
	0·667	0·79	
	0·756	0·7575	
	0·97		
2·54 cm	0·59		
	0·66		
	0·748		
	0·957		
3·81 cm	0·587		
	0·655		
	0·723		
	0·938		

TABLE 1

and
$$\rho_{ij} z_i^2 \left(\frac{\partial^2 \Psi}{\partial z^2}\right)_{ij} + 2z_i \left(\frac{\partial \Psi}{\partial z}\right)_{ij} + \left(\frac{\partial^2 \Psi}{\partial \alpha^2}\right)_{ij} - \cot \alpha_j \left(\frac{\partial \Psi}{\partial \rho}\right)_{ij} - z_i^2 \left(\frac{\partial \rho}{\partial z}\right)_{ij} \left(\frac{\partial \Psi}{\partial x}\right)_{ij} - \left(\frac{\partial \Psi}{\partial \alpha}\right)_{ij} \left(\frac{\partial \rho}{\partial \alpha}\right)_{ij} = 0. \quad (12)$$

The two sets of nonlinear algebraic equations (11) and (12) were solved by the Newton–Raphson technique. Solutions were obtained with $N = 1$ and $N = 2$, resulting in 5 and 10 term expansions respectively for Ψ and for ρ,

Once the solutions for Ψ and ρ have been obtained, the pressure distribution on the surface of the sphere in region D_1 can be calculated from

$$\frac{p}{p_\infty} = \left\{1 + \frac{\gamma - 1}{2} M_\infty^2 \left(1 - \left(\frac{v}{U_\infty}\right)^2\right)\right\}^{\gamma/(\gamma-1)}. \quad (13)$$

It was stated earlier that the parameter θ_{sh} was optimized by fitting calculated to experimental pressure distributions. For low Mach number supersonic flow $96^\circ < \theta_{sh} < 102^\circ$ was reported (Heberle *et al.* 1950); this was the range of θ_{sh} first tried. By fitting the predicted p/p_∞ to experimental data, we found that

$$\theta_{sh} = 91{\cdot}78 + 8{\cdot}59 M_\infty \quad (14)$$

results in the best fit for spheres when $0{\cdot}58 < M_\infty < 0{\cdot}97$.

3. Experiment

Pressure measurements were made on spheres of various sizes (1·02, 2·54 and 3·81 cm in diameter) and in various media (air, dry steam, wet steam) with free-stream Mach numbers ranging from 0·58 to 0·97. The three diameters were selected in order to investigate the Reynolds number effect. Because of space limitations and the blockage effect of the large spheres only the $D = 1{\cdot}02$ cm sphere was tested in wet steam.

Probe diameter	Reynolds number $M_\infty = 0{\cdot}58$	$M_\infty = 0{\cdot}66$	$M_\infty = 0{\cdot}75$	$M_\infty = 0{\cdot}95$
1·02 cm	$1{\cdot}18 \times 10^5$	$1{\cdot}3 \times 10^5$	$1{\cdot}5 \times 10^5$	$1{\cdot}78 \times 10^5$
2·54 cm	$2{\cdot}93 \times 10^5$	3.3×10^5	$3{\cdot}59 \times 10^5$	$4{\cdot}44 \times 10^5$
3·81 cm	$4{\cdot}33 \times 10^5$	$4{\cdot}63 \times 10^5$	$5{\cdot}07 \times 10^5$	$6{\cdot}34 \times 10^5$

TABLE 2

The 2·54 cm and 3·81 cm spheres were made from brass ball bearings. Two radially drilled holes were located at $\pm 30°$ to the central hole. The pressure lines were brought out through the 1·91 cm diameter support tube. The sphere was held in a holder mechanism in such a way that the plane of the three pressure measuring holes was horizontal and the sphere could be rotated about its vertical diameter. The angular position of the pressure holes could be determined to within $\pm 0{\cdot}5°$. Surface pressure readings up to $\pm 60°$ were obtained. Readings beyond 60° could not be obtained for these two spheres because of the interference of the support tube with the flow field. However the 1·02 cm diameter sphere did not suffer from this drawback because of the method of support.

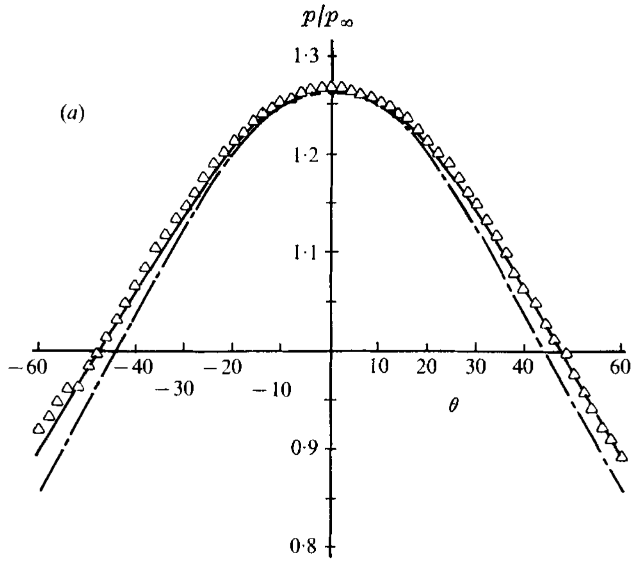
The smallest of the probes consists of a 1·02 cm diameter sphere with a 0·318 cm support tube inserted into and extending through it, perpendicular to the main flow direction. A 0·041 cm diameter radially drilled hole on the surface of sphere is used to measure the pressure. The hole is located along the probe centreline and passes through the centre of the sphere. This probe could be rotated through $\pm 180°$, and its angular position determined to within $\pm 0{\cdot}05°$.

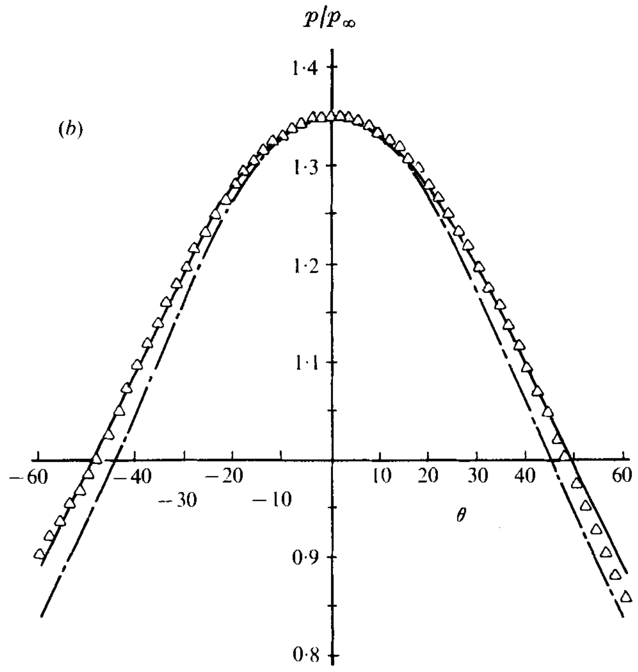
All the air data reported here were taken in a free air jet which was obtained by expanding air through a 10·16 cm ASME long radius flow nozzle to atmospheric pressure. The spheres were placed at a distance of 15·2 cm from the nozzle throat, at which position the diameter of the jet was 10·0 cm. The $D = 1{\cdot}02$ cm sphere was also tested in a steam tunnel of cross-section $5{\cdot}08 \times 30{\cdot}48$ cm². Further details of the free air jet, the steam test procedure and the calculation procedure for ascertaining the test flow conditions are given by Jaikrishnan (1976). A summary of the principal experimental conditions is shown in table 1. Typical Reynolds numbers for the air data in table 1 are given in table 2.

4. Results

A comparison of the experimental and the theoretical results has been made for all data indicated in table 1 in Jaikrishnan (1976). However, to be concise only a few representative results are shown here.

Figures 1 and 2 show the ratio p/p_∞ (surface pressure to stream static pressure) plotted against angular position θ. The data displayed on the first of these figures were taken in air with the $D = 2{\cdot}54$ cm probe. The data displayed in figure 2 were obtained in wet steam with the $D = 1{\cdot}02$ cm probe. These figures also contain calculated results for the first ($N = 1$) and second ($N = 2$) approximations to the power-series expressions for the theoretical solution. The figures show fairly good

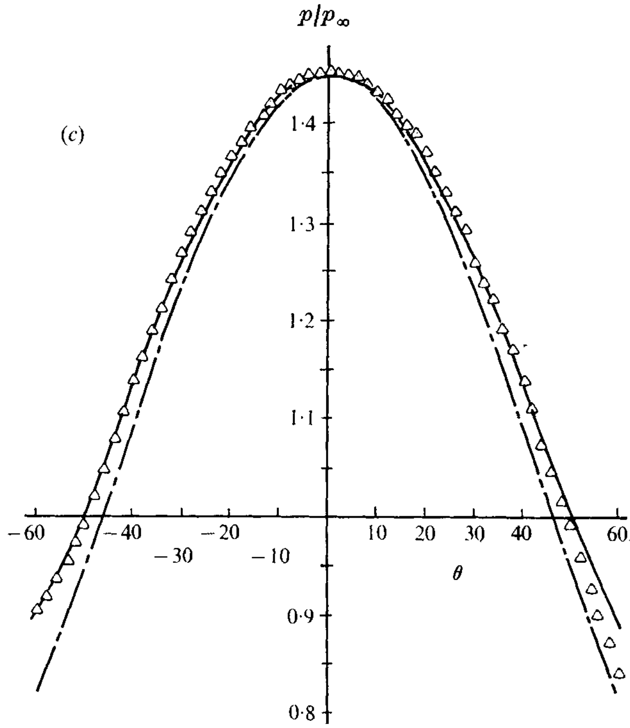




FIGURES 1 (*a*, *b*). For legend see facing page.

agreement between our predictions and experiments. These predictions were all obtained with shock angles calculated from the linear relationship (14). Unfortunately we were unable to obtain direct verification of this formula.

Figure 3 shows a comparison of the present solution, the solution of South & Jameson (1974) and the solution of Lipnitskii & Lifshits (1970) with our experimental data at $M_\infty = 0·8$. From this comparison it appears that shock fitting,



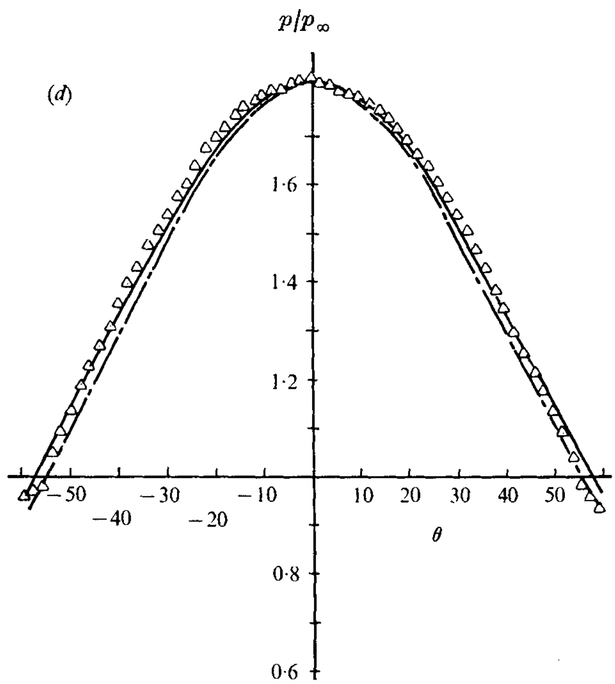
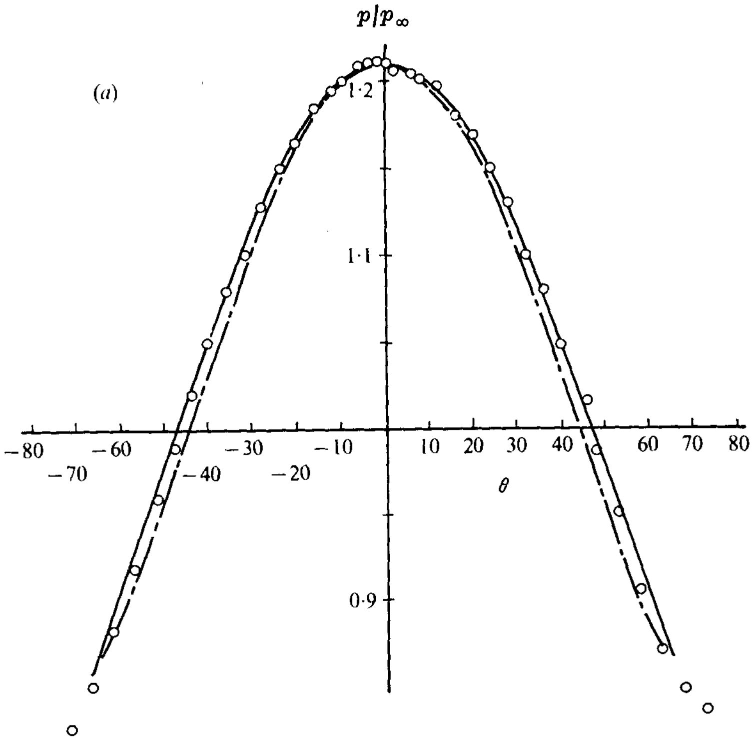
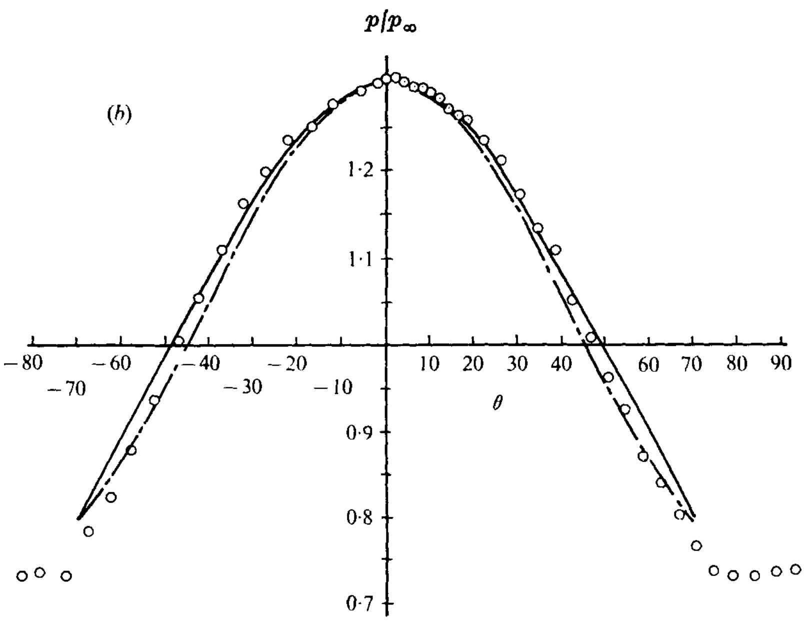


FIGURE 1. p/p_∞ *vs.* θ for 2·54 cm sphere, air data, Mach number: (*a*) $M_\infty = 0{\cdot}59$, (*b*) $M_\infty = 0{\cdot}666$, (*c*) $M_\infty = 0{\cdot}748$, (*d*) $M_\infty = 0{\cdot}957$. $\triangle$, experimental data; ——, $N = 2$; –·–, $N = 1$.





FIGURES 2 (*a*, *b*). For legend see facing page.

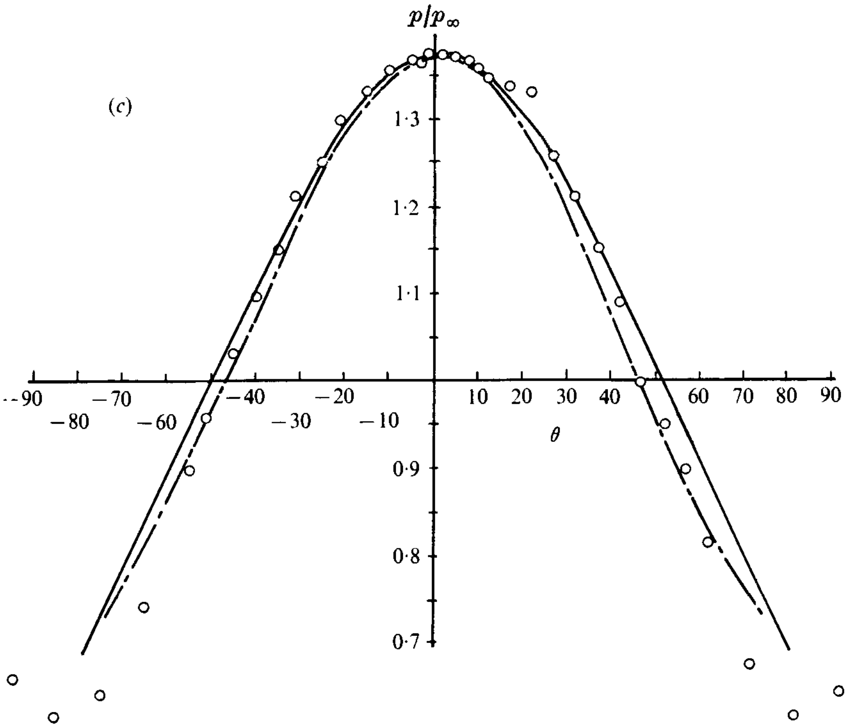


FIGURE 2. p/p_∞ *vs.* θ for 1·02 cm sphere, wet-steam data. Mach number: (*a*) $M_\infty = 0{\cdot}59$, (*b*) $M_\infty = 0{\cdot}69$, (*c*) $M_\infty = 0{\cdot}7575$. ○, experimental data; ——, $N = 2$; –·–, $N = 1$.

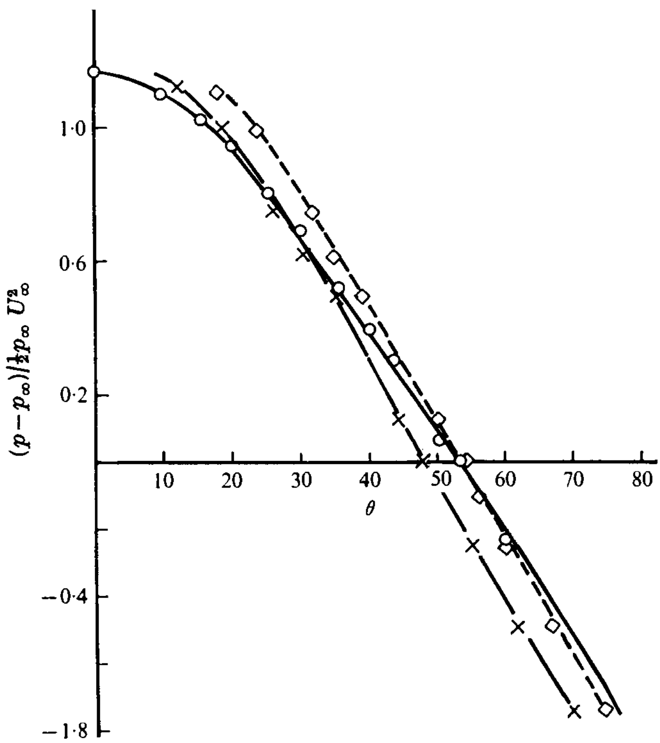
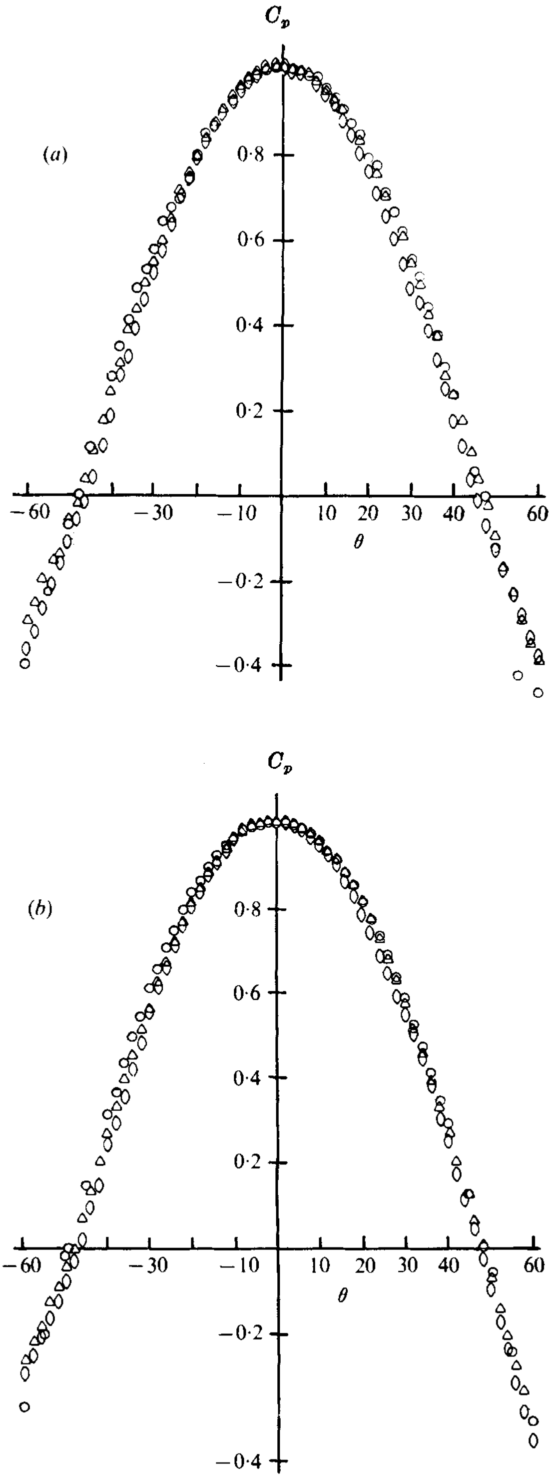


FIGURE 3. Comparison of various solutions. ——, authors' solution; –×–, South & Jameson (1974); –◇–, Lipnitskii & Lifshits (1970); –○–, authors' experimental data.



FIGURES 4 (*a*, *b*). For legend see facing page.

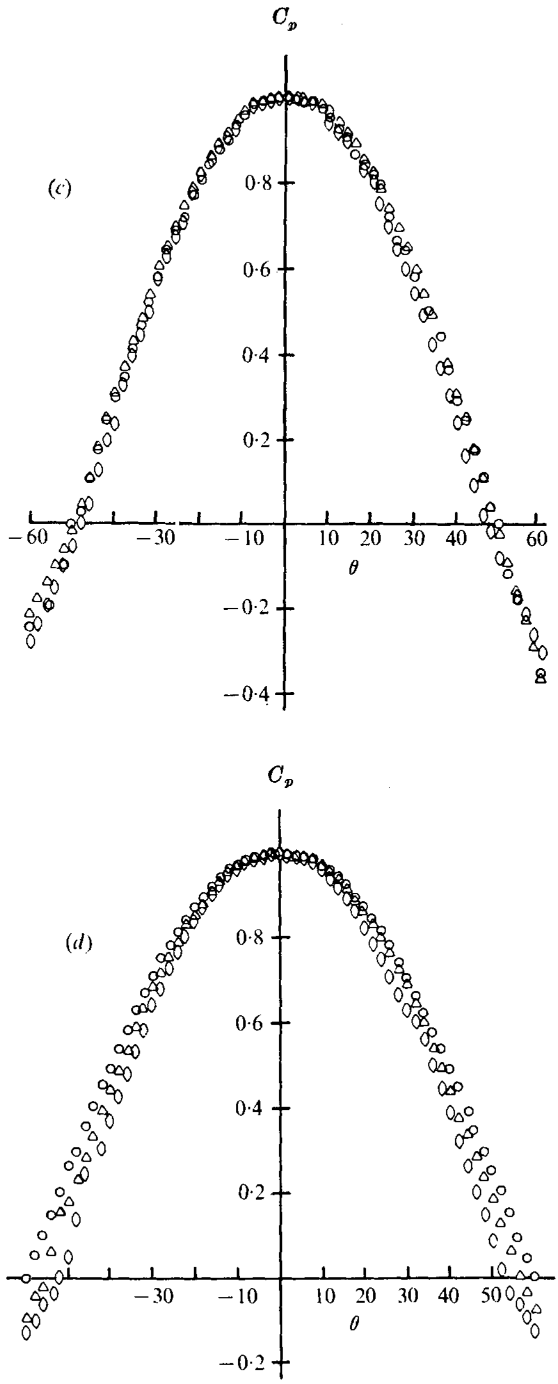


FIGURE 4. C_p *vs.* θ. Mach number: (*a*) $M_\infty = 0{\cdot}59$, (*b*) $M_\infty = 0{\cdot}66$, (*c*) $M_\infty = 0{\cdot}749$, (*d*) $M_\infty = 0{\cdot}96$. △, 2·54 cm sphere; ⬯, 3·81 cm sphere; ○, 1·02 cm sphere.

which is missing from the potential-flow solution of South & Jameson (1974), improves predictions considerably.

Although air data for the various spheres were taken at Mach numbers close to 0·58, 0·66, 0·75 and 0·95, the Mach numbers of the various runs were not exactly the same. In order to study the effect of the Reynolds number on the flow field, the normalized pressure coefficient $C_p = (p-p_\infty)/(p_0-p_\infty)$ was calculated and is plotted against angular position in figure 4. At the lower Mach numbers the experimental points seem to fall on the same curve, indicating no Reynolds number effect in the domain $1{\cdot}18\times10^5 < Re < 5{\cdot}07\times10^5, 0{\cdot}58 < M < 0{\cdot}75$. This is in substantial agreement with conclusions reached by Naumann (1953). At the highest Mach number tested, however, separation of experimental points according to the Reynolds number is indicated in figure 4(*d*). No such effect could be shown theoretically and we suspect that stem blockage is responsible for the Reynolds number effect in this flow regime. Another contributing cause might be deviation from parallel flow conditions, resulting from distortion of the flow field owing to the presence of the larger spheres.

The experimental work described in this paper was performed at the Westinghouse Research Laboratories, Pittsburgh. The authors thank Westinghouse Research Laboratories for permission to use their test facilities and in particular Dr W. A. Stewart, Manager, Heat Transfer and Fluid Dynamics, for his interest in this work.

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